OFDM mmWave Channel Estimation with OMP
Wuqiong Zhao

This is a simple simulation of millimeter wave (mmWave) channel estimation in wideband assisted by OFDM with orthogonal matching pursuit (OMP) algorithm. The main idea of OFDM mmWave channel estimation is the shared angle of arrival (AoA) and angle of departure (AoD). However, in wideband mmWave MIMO systems, the beam squint effect cannot be neglected. For simplicity, this effect is not considered in this simulation. The number in the bracket after OMP is the number of carriers used to estimate the AoA and AoD (i.e. non-zero elements in the beam domain). The R suffix means re-estimating the carriers used to estimate the AoA and AoD using least square (LS) after the support is calculated.

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1 System Settings

The simulation adopts the geometric channel model for millimeter wave (mmWave).

<table>
<thead>
<tr>
<th>Name</th>
<th>Antenna Number</th>
<th>Beam Number</th>
<th>Grid Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter</td>
<td>8 × 1</td>
<td>2 × 1</td>
<td>8 × 1</td>
</tr>
<tr>
<td>Receiver</td>
<td>16 × 1</td>
<td>4 × 1</td>
<td>16 × 1</td>
</tr>
</tbody>
</table>

- Channel Sparsity: 6;
- Off Grid Effect: false;
- Bandwidth: Wideband;
- Carriers: 64.
2 Simulation Results

2.1 NMSE v.s. SNR (Pilot: 16)

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>OMP (1)</th>
<th>OMP (2)</th>
<th>OMP (4)</th>
<th>OMP (8)</th>
<th>OMP (8R)</th>
<th>OMP (All)</th>
<th>Oracle LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>−10</td>
<td>3.47</td>
<td>3.63</td>
<td>3.72</td>
<td>3.87</td>
<td>3.42</td>
<td>6.55</td>
<td>1.03</td>
</tr>
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<td>−8</td>
<td>1.88</td>
<td>1.92</td>
<td>1.87</td>
<td>1.77</td>
<td>1.34</td>
<td>4.27</td>
<td>−1.50</td>
</tr>
<tr>
<td>−6</td>
<td>0.93</td>
<td>0.79</td>
<td>0.50</td>
<td>0.29</td>
<td>−0.14</td>
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<td>−3.18</td>
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<tr>
<td>−4</td>
<td>−0.10</td>
<td>−0.57</td>
<td>−1.15</td>
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<td>−2.22</td>
<td>1.02</td>
<td>−5.25</td>
</tr>
<tr>
<td>−2</td>
<td>−1.22</td>
<td>−2.17</td>
<td>−3.51</td>
<td>−4.59</td>
<td>−5.43</td>
<td>−1.13</td>
<td>−7.41</td>
</tr>
<tr>
<td>0</td>
<td>−1.81</td>
<td>−3.18</td>
<td>−4.78</td>
<td>−5.65</td>
<td>−6.39</td>
<td>−2.53</td>
<td>−9.10</td>
</tr>
<tr>
<td>2</td>
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<td>−5.32</td>
<td>−7.33</td>
<td>−8.65</td>
<td>−9.58</td>
<td>−4.99</td>
<td>−10.75</td>
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<tr>
<td>4</td>
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<td>−6.69</td>
<td>−9.50</td>
<td>−10.78</td>
<td>−11.94</td>
<td>−6.61</td>
<td>−12.65</td>
</tr>
<tr>
<td>6</td>
<td>−5.72</td>
<td>−9.37</td>
<td>−12.69</td>
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<td>−14.44</td>
<td>−9.48</td>
<td>−14.75</td>
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<tr>
<td>8</td>
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<td>−11.70</td>
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<td>−18.50</td>
<td>−18.96</td>
<td>−20.03</td>
<td>−15.08</td>
<td>−20.34</td>
</tr>
<tr>
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<td>−14.43</td>
<td>−21.23</td>
<td>−23.15</td>
<td>−23.00</td>
<td>−23.26</td>
<td>−21.45</td>
<td>−23.45</td>
</tr>
<tr>
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<td>−15.13</td>
<td>−23.63</td>
<td>−24.71</td>
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<td>−28.03</td>
<td>−29.41</td>
<td>−23.36</td>
<td>−29.54</td>
</tr>
</tbody>
</table>

Simulated with 500 Monte Carlo tests.
2.2 NMSE v.s. Pilot (SNR: 0 dB)

<table>
<thead>
<tr>
<th>Pilot</th>
<th>OMP (1)</th>
<th>OMP (2)</th>
<th>OMP (4)</th>
<th>OMP (8)</th>
<th>OMP (8R)</th>
<th>OMP (All)</th>
<th>Oracle LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.84</td>
<td>0.72</td>
<td>0.50</td>
<td>0.28</td>
<td>0.00</td>
<td>1.96</td>
<td>-3.42</td>
</tr>
<tr>
<td>8</td>
<td>0.20</td>
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<td>-0.67</td>
<td>-1.11</td>
<td>-1.53</td>
<td>1.06</td>
<td>-5.34</td>
</tr>
<tr>
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<td>-1.73</td>
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<td>-6.55</td>
</tr>
<tr>
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<td>-2.20</td>
<td>-3.47</td>
<td>-4.49</td>
<td>-5.28</td>
<td>-1.18</td>
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<td>-5.42</td>
<td>-6.24</td>
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<td>16</td>
<td>-2.26</td>
<td>-3.91</td>
<td>-5.73</td>
<td>-6.84</td>
<td>-7.72</td>
<td>-3.29</td>
<td>-9.03</td>
</tr>
<tr>
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<td>-4.45</td>
<td>-6.63</td>
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</tr>
</tbody>
</table>

Simulated with 500 Monte Carlo tests.
### 2.3 NMSE v.s. Pilot (SNR: 10 dB)

<table>
<thead>
<tr>
<th>Pilot</th>
<th>OMP (1)</th>
<th>OMP (2)</th>
<th>OMP (4)</th>
<th>OMP (8)</th>
<th>OMP (8R)</th>
<th>OMP (All)</th>
<th>Oracle LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.05</td>
<td>1.85</td>
<td>2.77</td>
<td>3.43</td>
<td>3.79</td>
<td>1.65</td>
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<td>9.72</td>
<td>5.61</td>
<td>15.50</td>
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<td>8.44</td>
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<td>7.22</td>
<td>11.74</td>
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<td>17.33</td>
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<td>21.84</td>
<td>21.68</td>
<td>21.95</td>
<td>20.08</td>
<td>22.11</td>
</tr>
</tbody>
</table>

Simulated with 500 Monte Carlo tests.
3 Simulation Configuration

3.1 Configuration File

Listing 1: MIMO_wideband.sim

```
# MIMO_wideband.sim
# Wideband (OFDM) mmWave Channel Estimation with OMP
# Author: Wuqiong Zhao
# Date: 2022-09-26

version: 0.1.0 # the targeted mmCEsim version
meta: # document meta data
  title: OFDM mmWave Channel Estimation with OMP
  description:
    This is a simple simulation of millimeter wave (mmWave) channel estimation in wideband assisted by OFDM with orthogonal matching pursuit (OMP) algorithm.
    The main idea of OFDM mmWave channel estimation is the shared angle of arrival (AoA) and angle of departure (AoD). However, in wideband mmWave MIMO systems, the beam squint effect cannot be neglected. For simplicity, this effect is not considered in this simulation.
    The number in the bracket after OMP is the number of carriers used to estimate the AoA and AoD (i.e. non-zero elements in the beam domain). The R suffix means re-estimating the carriers used to estimate the AoA and AoD using least square (LS) after the support is calculated.
  author: Wuqiong Zhao
  email: contact@mmcesim.org
  website: https://mmcesim.org
  license: MIT
  date: "2022-09-16"
  comments: This is an uplink channel.

physics:
  frequency: wide # assume narrow band
  carriers: 64
  off_grid: false # do not consider off-grid problem

nodes:
  - id: BS # this should be unique
    role: receiver
    num: 1 # this is the default value
    size: [16, 1] # ULA with size 16x1
    beam: [4, 1]
    grid: same # the same as physics size
    beamforming:
      variable: "W"
      scheme: random
  - id: UE # user
    role: transmitter
    num: 1 # a single-user model
    size: 8 # ULA with size 8
    beam: 2
    grid: 8
    beamforming:
      variable: "F"
      scheme: random

channels:
  - id: H
    from: BS
to: UE # 'from -> to' specifies the channel direction
sparsity: 6
gains:
```
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mode: normal
mean: 0
variance: 1
sounding:
variables:
  received: "Y" # received signal vector
  noise: "noise" # received noise vector
channel: "H_cascaded" # the cascaded channel (actually the same as 'H' for simple MIMO)
macro:
- name: OFDM_ANGLE_EST_NUM
  value: 4
in_alg: true
- name: OFDM_RE_ESTIMATE
  value: false
in_alg: true
- name: SPARSITY_EST
  value: 6
in_alg: true
preamble: |
  COMMENT Here starts the preamble.
estimation: |
  VNt ::m = NEW `DICTIONARY.T`
  VNr ::m = NEW `DICTIONARY.R`
  lambda_hat = INIT `GRID.*`
  H_hat = INIT `SIZE.R` `SIZE.T` `CARRIERS_NUM`
  Q = INIT `MEASUREMENT` `GRID.*`
  i::u0 = LOOP 0 `PILOT` `/ BEAM.T`
  F_t ::m = NEW F_(:, :, i)
  W_t ::m = NEW W_(:, :, i)
  Q_{i*BEAM.*:(i+1)*BEAM.*-1,:} = \kron(F_t^T, W_t^H) \kron(VNt^*, VNr) # the sensing matrix
  END
BRANCH
angle_est = INIT `GRID.R` `GRID.T` dtype=f
k::u0 = LOOP 0 `OFDM_ANGLE_EST_NUM`
  none_zero :: u1 = NEW \find(\abs(VNr H_cascaded_ {:,:,k}@VNt)>0.1)
  lambda_hat = ESTIMATE Q Y_{{:,k}} none_zero
  angle_est = angle_est + \pow(\abs(lambda_hat), 2)
  IF ! `OFDM_RE_ESTIMATE`
    H_hat_ {:,:,k} = VNr \kron(\reshape(lambda_hat, `GRID.R`, `GRID.T`) @ VNt^H
  END
END
ranking :: u1 = NEW \sort_index(-angle_est)
support :: u1 = NEW ranking_ {0 : `SPARSITY_EST` -1}
index_start :: u0 = NEW 0
  IF ! `OFDM_RE_ESTIMATE`
    index_start = `OFDM_ANGLE_EST_NUM`
  END
k::u0 = LOOP index_start `CARRIERS_NUM`
  lambda_hat = CALL LS_support Q Y_{{:,k}} support
  H_hat_ {:,:,k} = VNr \kron(\reshape(lambda_hat, `GRID.R`, `GRID.T`) @ VNt^H
  END
RECOVER H_hat
MERGE
conclusion: |
  PRINT "" >> `JOB_CNT`+1 `'/ `JOB_NUM` 'n'
simulation: |
  backend: cpp # cpp (default) | matlab | octave | py
  metric: [NMSE] # used for compare
  jobs:
    - name: "NMSE v.s. SNR (Pilot: 16)"
      test_num: 500
      SNR: [-10:2:20]
SNR_mode: dB # dB (default) | linear
pilot: 16
  # pilot_mode: percent # num (default) | percent
algorithms: # compare different languages
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 1
    label: OMP (1) # used in report
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 2
    label: OMP (2) # used in report
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 4
    label: OMP (4) # used in report
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 8
    label: OMP (8) # used in report
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 64
    label: OMP (All) # used in report
  - alg: Oracle_LS
    label: Oracle LS
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 64
  - name: "NMSE v.s. Pilot (SNR: 0 dB)"
test_num: 500
SNR: 0
pilot: [6:2:32]
  # pilot_mode: percent # num (default) | percent
algorithms: # compare different languages
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 1
    label: OMP (1) # used in report
  - alg: OMP
    max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 2
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label: OMP (2) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 4
label: OMP (4) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 8
label: OMP (8) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 8
    - name: OFDM_RE_ESTIMATE
      value: true
label: OMP (8R) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 64
label: OMP ( All ) # used in report
- alg: Oracle_LS
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 64
label: Oracle LS
- name: "NMSE v.s. Pilot (SNR: 10 dB)"
test_num: 500
SNR: 10
pilot: [6:2:32]
# pilot_mode: percent # num (default) | percent
algorithms: # compare different languages
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 1
label: OMP (1) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 2
label: OMP (2) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 4
label: OMP (4) # used in report
- alg: OMP
  max_iter: 6
  macro:
    - name: OFDM_ANGLE_EST_NUM
      value: 8
label: OMP (8) # used in report
- alg: OMP
  max_iter: 6
  max_iter: 6
  max_iter: 6
3.2 Algorithms

Listing 2: OMP.a.lg

```plaintext
#! Function: OMP
#! Description: Orthogonal matching pursuit compressed sensing.
#! Author: Wuqiong Zhao
#! Date: 2022-09-16
#! Version: 0.1.0

# Input:
# - Q: Sensing matrix
# - y: Received signal
# - L: Sparsity

# Output:
# - h: The estimated sparse signal

h::v = FUNCTION OMP Q::m y::v L::u0
COMMENT Start of OMP algorithm!

h = \zeros(\size(Q, 1)) # initialize as zeros
Q_H ::m = NEW Q^H # the conjugate transpose of Q
r = NEW y # residual
r_last ::v = NEW r * 2 # the residual in last iteration
support = INIT \length (y) dtype =u # over-length support array
term = INIT $ \size(Q_H, 0)$ dtype =f # float number array
j:: u0 = NEW 0
a::v = INIT
FOR $j != \length(y)$ $j = j + 1$
    term = \abs (Q_H @ r)
    index :: u0 = NEW \index_max (term)
    IF j \&\& \ismember (index, support_{0:j-1})
        BREAK # end of the LOOP
    END
    support_{j} = index
    columns ::m = NEW Q_{:, support_{0:j}}
a = \solve (columns, y)
r = y - columns @ a
    IF \sum (\abs (r - r_last)) / \sum (\abs(r_last)) < 0.0001 || j + 1 >= L
        break # end of the loop
    END
```

---

```plaintext
macro:
- name: OFDM_ANGLE_EST_NUM
  value: 8
- name: OFDM_RE_ESTIMATE
  value: true
- label: OMP (8R) # used in report
- alg: OMP
  max_iter: 6
macro:
- name: OFDM_ANGLE_EST_NUM
  value: 64
- label: OMP (All) # used in report
- alg: Oracle_LS
  label: Oracle LS
macro:
- name: OFDM_ANGLE_EST_NUM
  value: 64
report:
name: OFDM_mmWave_CE_OMP_Simulation
format: [pdf , latex] # both compiled PDF and tex files
plot: true # plot data
table: false # do not print table
latex:
command: pdflatex # command to compile the report
UTF8: false # no need for UTF8 support with this setting
```
4 mmCesim Information

This report is auto generated by mmCesim. The application mmCesim is a powerful tool to simulate millimeter wave (mmWave) channel estimation (CE) for both experts and learners.

mmCesim is open source! The software can be freely used and distributed under the MIT license.

- Official Website: https://mmcesim.org
- Documentation: https://mmcesim.org/doc
- Tutorial: https://mmcesim.org/tutorial
- Examples: https://mmcesim.org/example
- Web Application: https://app.mmcesim.org
- Blog: https://blog.mmcesim.org
- Publications: https://pub.mmcesim.org
- GitHub Organization: https://github.com/mmcesim
- Twitter: https://twitter.com/mmcesim